**PART 1**

The Master Theorem applies to recurrences of the following form:

T (n) = aT (n/b) + f(n) where a ≥ 1 and b > 1 are constants and f(n) is an asymptotically positive function.

There are 3 cases:

1. If f(n) = O( ) for some constant > 0, then T (n) = Θ().

2. If f(n) = Θ \* where k ≥ 0, then T (n) = Θ( \* logn).

3. If f(n) = Ω( ) where > 0,

and f(n) satisfies the regularity condition, then T (n) = Θ(f(n)).

Regularity condition: af(n/b) ≤ cf(n) for some constant c < 1 and all sufficiently large n.

**limitations of master theorem:**

1. a must be constant

2. a must be greater than or equal to 1

3. b must be greater than 1

4. f(n) should be positive function

Ex: T(n) = 2T(n/2) - n2

here f(n) = -n2

Hence cannot apply master theorem

5. f(n) must be polynomially (not logarithmically greater) greater than

Ex 1: T(n) = 2T(n/2) + n(log n)

= = n

condition: f(n) >

n(log n) > n

This is logarithmically greater. i.e., if log n does not exist then n is not > n

Hence, we cannot apply the master's theorem.

Ex 2: T(n) = 2T(n/2) + (n2)(log n)

= = n

condition: f(n) >

(n2)(log n) > n

this is polynomially greater. i.e., if log n does not exist then n2 is still > n

Hence, we can apply the master's theorem.

Find the time complexity of the below functions in Θ form. Write NA if the function does not apply to any case.

1. T (n) = 3T (n/2) + n

T (n) = Θ() (Case 1)

1. T (n) = 64T (n/8) −

N.A.(see point 4 in limitations)

1. T (n) = 2nT (n/2) +

N.A.(see point 1 in limitations)

1. T (n) = 3T (n/3) + n/2

T (n) = Θ(n log n) (Case 2)

1. T (n) = 7T (n/3) +

T (n) = Θ() (Case 3)